Study Guide Review

ASSESSMENT AND INTERVENTION

Personal Math Trainer

Assign or customize module reviews.

MODULE PERFORMANCE TASK

COMMON CORE

Mathematical Practices: MP.1, MP.2, MP.5, MP.7, MP.8
F-BF.A.1a, F-IF.A.3

SUPPORTING STUDENT REASONING

Students may be completely baffled by the challenge before them. Here are some questions they might have.

- **Can I just write the numbers in a very long column and then add them normally?** Of course you can. How quickly do you think you can do that?
- **Can I add the numbers in groups of 10, and then find the sum of all of the groups?** Yes, this is a more efficient method than adding a list of 100 numbers.

Essential Question: How are patterns and sequences used to solve real-world problems?

Key Vocabulary
arithmetic sequence *(sucesión aritmética)*
common difference *(diferencia común)*
explicit rule *(fórmula explícita)*
recursive rule *(fórmula recurrente)*
sequence *(sucesión)*
term *(término)*

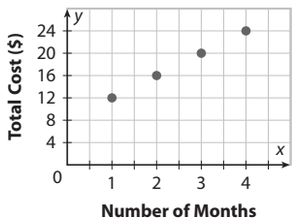
KEY EXAMPLE (Lesson 4.1)

A software subscription is $4 a month plus a start-up fee of $8. Use the explicit rule $f(n) = 4n + 8$. Construct and graph the first 4 terms of the sequence described.

n	1	2	3	4
$f(n)$	12	16	20	24

Represent the sequence in a table.

(1, 12), (2, 16), (3, 20), (4, 24) *Generate ordered pairs.*



KEY EXAMPLE (Lesson 4.2)

Write a recursive rule and an explicit rule for the sequence 20, 14, 8, 2 ….

$f(1) = 20, d = 14 - 20 = -6$ *Find the first term and common difference.*

Given $f(1), f(n) = f(n-1) + d$ for $n \geq 2$. *Use the general form of the recursive rule.*

Recursive Rule: $f(1) = 20, f(n) = f(n-1) - 6$

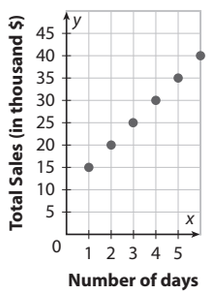
$f(n) = f(1) + d(n-1)$ *Use the general form of the explicit rule.*

Explicit Rule: $f(n) = 20 - 6(n-1)$

KEY EXAMPLE (Lesson 4.3)

Construct an explicit rule in function notation for the arithmetic sequence represented in the graph, and use it to solve the problem.

The graph shows the total predicted sales $f(n)$ for the next n days at a clothing store. What are the total predicted sales on day 10?



15, 20, 25, 30… *Write a sequence to represent the information.*

$d = 20 - 15 = 5$ *Find the common difference.*

$f(n) = f(1) + d(n-1)$ *Use the general explicit rule.*

$f(n) = 15 + 5(n-1)$

$f(10) = 15 + 5(10-1)$ *Find $f(10)$.*

$f(10) = 60$

The total predicted sales on day 10 are $60,000.

SCAFFOLDING SUPPORT

- For students needing more structure, show them the expression Gauss used to solve this problem: $\frac{n(n+1)}{2}$. Ask students to explain why it works.
- Challenge students who finish early to write an expression that can be used to find the sum of consecutive numbers from 1 to n for any value of n.

EXERCISES

Write the first 4 terms of each sequence following the given rule. *(Lesson 4.1)*

1. $f(n) = n^2 - 4$

−3, 0, 5, 12

2. $f(1) = -12, f(n) = 2f(n-1)$

−12, −24, −48, −96

Determine if each of the following sequences is arithmetic. If so, write a recursive rule and an explicit rule for the sequence. If not, explain why. *(Lesson 4.2)*

3. −8, −1, 6, 13…

$f(1) = -8, f(n) = f(n-1) + 7; f(n) = -8 + 7(n-1)$

4. 1, 8, 27, 81…

Not an arithmetic sequence, because there is no common difference.

5. The table below shows the balance of a savings account each month after being opened. The balance can be represented with an arithmetic sequence. Write an explicit rule and a recursive rule for the sequence. What will the account balance be after 10 months? *(Lesson 4.3)*

Time (months)	1	2	3	4
Balance ($)	750	715	680	645

$f(n) = 750 - 35(n-1); f(1) = 750; f(n) = f(n-1) - 35$; The balance will be $435 after 10 months.

MODULE PERFORMANCE TASK

There Has to Be an Easier Way

Quick, now, what's the sum: $1 + 2$?

Okay, you got that one. How about this: $1 + 2 + 3$?

You're really sailing along! Okay, how about this one: $1 + 2 + 3 + \ldots + 98 + 99 + 100$?

Whoops. That's the problem that mathematician Carl Friedrich Gauss solved quickly when he was 10 years old. And that's the problem you're being asked to solve now. Getting the right answer isn't as important as coming up with some interesting observations about the problem or some ideas that might lead you in the direction of the right answer.

Gauss was 10 years old in 1787, so he didn't have a calculator! No calculator for you either—just use your own paper to work on the task. Then use numbers, words, pictures, or algebra to explain how you reached your conclusion.

DISCUSSION OPPORTUNITIES

- Ask students to share other ways they could have grouped numbers to add them more efficiently than adding a list of 100 numbers. Compare different methods.
- Have students share how they overcame their "roadblocks" in solving this problem.

SAMPLE SOLUTION

$$\begin{array}{rrrrrrrrrrr} 1 + & 2 + & 3 + \ldots + & 98 + & 99 + 100 \\ 100 + & 99 + & 98 + \ldots + & 3 + & 2 + & 1 \\ \hline 101 + & 101 + & 101 + \ldots + & 101 + & 101 + 101 \end{array}$$

If you add the whole numbers from 1 to 100 *twice*, the sum is $100 \times 101 = 10{,}100$. So the sum of the whole numbers from 1 to 100 *once* is $10{,}100 \div 2 = 5{,}050$.

Assessment Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain.

0 points: Student does not demonstrate understanding of the problem.

Ready to Go On?

ASSESS MASTERY

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

ASSESSMENT AND INTERVENTION

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

ADDITIONAL RESOURCES

Response to Intervention Resources

- Reteach Worksheets

Differentiated Instruction Resources

- Reading Strategies EL
- Success for English Learners EL
- Challenge Worksheets

Assessment Resources

- Leveled Module Quizzes

Ready to Go On?

4.1–4.3 Patterns and Sequences

- Online Homework
- Hints and Help
- Extra Practice

Write the first 4 terms of each sequence defined by the rule given. *(Lesson 4.1)*

1. $f(1) = 8, f(n) = f(n-1) - 4$

8, 4, 0, −4

2. $f(n) = \frac{n^2}{2}$

0.5, 2, 4.5, 8

Write a recursive rule and an explicit rule for each arithmetic sequence. Then, find the 20th term of each sequence. *(Lessons 4.2, 4.3)*

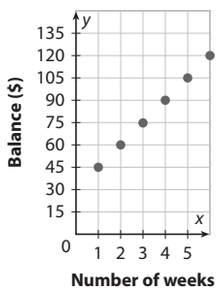
3. 2, 0, −2, −4…

$f(1) = 2, f(n) = f(n-1) - 2; f(n) = 2 - 2(n-1); f(20) = -36$

4. 45, 55, 65, 75…

$f(1) = 45, f(n) = f(n-1) + 10; f(n) = 45 + 10(n-1); f(20) = 235$

5. Each Saturday, Tina mows lawns to earn extra money which she puts into a savings account. The graph shows the balance of Tina's savings account over the first six weeks of mowing lawns. Write an explicit function to describe this sequence. According to this pattern, how much will Tina have in her account after 15 weeks of mowing lawns? *(Lesson 4.3)*



$f(n) = 45 + 15(n-1)$; She will have $255 after 15 weeks.

ESSENTIAL QUESTION

6. What are two ways of representing an arithmetic sequence?

Possible Answer: An arithmetic sequence can be represented by a recursive rule which gives the first term and defines the *n*th term by relating it to the previous term, or by an explicit rule which defines the *n*th term as a function of *n*.

COMMON CORE Common Core Standards

Lesson	*Items*	Content Standards	Mathematical Practices
4.1	1	**F-IF.A.3**	**MP.2**
4.1	2	**F-IF.A.3**	**MP.2**
4.2	3	**F-BF.A.2, F-LE.A.2, F-IF.A.3**	**MP.7**
4.2	4	**F-BF.A.2, F-LE.A.2, F-IF.A.3**	**MP.7**
4.3	5	**F-BF.A.1a, F-LE.A.2, F-IF.A.3**	**MP.4**

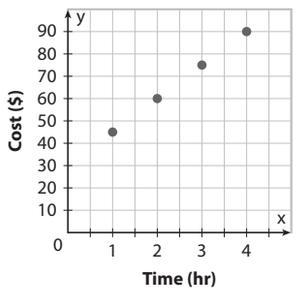
MODULE 4
MIXED REVIEW

Assessment Readiness

1. Consider a sequence defined by the recursive rule $f(1) = 15; f(n) = f(n-1) - 6$ for $n \geq 2$. Choose True or False for each statement.

A. The second term of the sequence is 8. ○ True ● False
B. The third term of the sequence is 3. ● True ○ False
C. The fourth term of the sequence is −3. ● True ○ False

2. The cost of renting a moped for 1, 2, 3, or 4 hours and can be represented by an arithmetic sequence. The base fee is $30, and the cost per hour is $15. The graph shows the sequence. Choose True or False for each statement.



A. The domain of the sequence is $\{1, 2, 3, 4\}$. ● True ○ False
B. The range of the sequence is the set of all real numbers. ○ True ● False
C. An explicit rule for the sequence is $f(n) = 15 + 30n$. ○ True ● False

3. Look at each possible solution of the inequality $-12 < 2x + 8 < -6$ below. Is the value of x actually a solution of the inequality? Select Yes or No for each value of x.

A. $x = -10$ ○ True ● False
B. $x = -8$ ● True ○ False
C. $x = -14$ ○ True ● False

4. On Monday, Mr. Sanchez started reading a 225-page biography. He plans to read 15 pages each day until he finishes the book. Write an explicit function to represent the number of pages he has left to read depending on the day number; Monday is day number 1, Tuesday is day number 2, and so on. Find f(5) and interpret its meaning in this situation.

***f*(*n*) = 225 − 15(*n* − 1); *f*(5) = 165**

On Friday, he'll have 165 pages left to read.

COMMON CORE Common Core Standards

Lesson	Items	Content Standards	Mathematical Practices
4.1	1	**F-IF.A.3**	**MP.2**
4.2	2	**F-IF.A.1, F-BF.A.2**	**MP.4**
2.5	3*	**A-REI.B.3**	**MP.2**
4.3	4	**F-BF.A.1a**	**MP.4**

* Item integrates mixed review concepts from previous modules or a previous course.

MIXED REVIEW

Assessment Readiness

ASSESSMENT AND INTERVENTION

Assign ready-made or customized practice tests to prepare students for high-stakes tests.

ADDITIONAL RESOURCES

Assessment Resources

- Leveled Module Quizzes: Modified, B

AVOID COMMON ERRORS

Item 1 Some students get recursive and explicit rules confused, and they will use the position number n to substitute for $f(n-1)$ in a recursive rule rather than use the previous term of the sequence. Emphasize that since the notation $f(n-1)$ is for the *output* of a function, they cannot substitute the position number.