

## Properties of Linear Functions

Rather than simply comparing graphs of two functions in the same family, we can also compare functions expressed in different ways.

Ex. 1: Let's find out key information about the functions below, so that we can compare them:

<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>2</td> </tr> <tr> <td>3</td> <td>5</td> </tr> <tr> <td>4</td> <td>8</td> </tr> <tr> <td>5</td> <td>11</td> </tr> </tbody> </table>	x	f(x)	2	2	3	5	4	8	5	11	$f(x) = 3x - 4$ Domain: All <u>real number</u> such that $2 \leq x \leq 5$ includes decimals!	<table border="1"> <thead> <tr> <th>x</th> <th>g(x)</th> </tr> </thead> <tbody> <tr> <td>2</td> <td><math>3\frac{1}{3}</math></td> </tr> <tr> <td>3</td> <td>4</td> </tr> <tr> <td>4</td> <td><math>4\frac{2}{3}</math></td> </tr> <tr> <td>5</td> <td><math>5\frac{1}{3}</math></td> </tr> </tbody> </table>	x	g(x)	2	$3\frac{1}{3}$	3	4	4	$4\frac{2}{3}$	5	$5\frac{1}{3}$	$g(x) = \frac{2}{3}x + 2$ Domain: All real numbers such that $2 \leq x \leq 5$
x	f(x)																						
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3	5																						
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x	g(x)																						
2	$3\frac{1}{3}$																						
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What is initial value for f(x)? 2		What is initial value for g(x)? $3\frac{1}{3}$																					
What is the final value of f(x)? 11		What is the final value of g(x)? $5\frac{1}{3}$																					
What is the range of f(x)? $2 \leq f(x) \leq 11$		What is the range of g(x)? $3\frac{1}{3} \leq g(x) \leq 5\frac{1}{3}$																					
What is the slope of f(x)? 3		What is the slope of g(x)? $\frac{2}{3}$																					

Now, compare f(x) and g(x)...

Compare the initial values: The initial value of f(x) is 2, which is less than the initial value of g(x),  $3\frac{1}{3}$

Compare the range: The range of f(x) is  $2 \leq f(x) \leq 11$ , while the range of g(x) is  $3\frac{1}{3} \leq g(x) \leq 5\frac{1}{3}$

Compare the slopes: The slope of f(x) is 3, which is more than the slope of g(x),  $\frac{2}{3}$

Ex. 2: f(x) is defined by the function  $f(x) = 2x + 9$  and has a domain of all real numbers such that  $-4 \leq x \leq -1$ . The table show some ordered pairs of g(x), which has the same domain as f(x).

x	g(x)
-4	10
-3	9
-2	8
-1	7

Compare the initial value, range, and rate of change of f(x) and g(x).

$$f(-4) = 2(-4) + 9 = -8 + 9 = 1 \rightarrow \text{initial value}$$

$$\text{Range: } 1 \leq f(x) \leq 7 \quad 7 \leq g(x) \leq 10$$

$$\text{Rate of change: } f(x): 2$$

$$g(x): \frac{9-10}{-3+4} = \frac{-1}{1} = -1$$

$$f(-1) = 2(-1) + 9 = -2 + 9 = 7 \rightarrow \text{ending value}$$

In order to compare properties of linear functions given a graph and a description, it is often helpful to write a rule for the functions.

Ex. 3: One group of hikers hiked at a steady rate of 6.5 km per hour for 4 hours. The graph shows the distance a second group of hikers hiked in kilometers.

Write a function to represent each, then compare the initial value, domain, range, and slopes of each.

$$f(x) = 6.5x$$

$$D: 0 \leq x \leq 4$$

$$f(0) = 6.5(0) = 0$$

$$f(4) = 6.5(4) = 26$$

$$R: 0 \leq f(x) \leq 26$$

$$m = 6.5$$

$$g(x): \frac{36-0}{4.5-0} = 8$$

$$g(x) = 8x$$

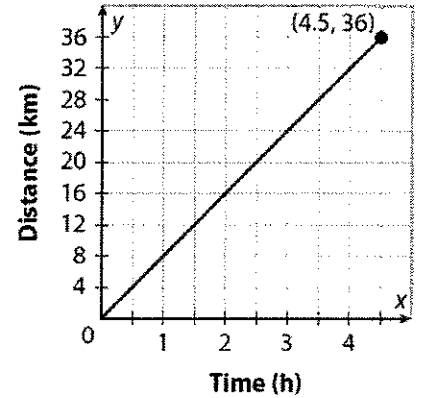
$$D: 0 \leq x \leq 4.5$$

$$g(0) = 8(0) = 0$$

$$g(4.5) = 36$$

$$R: 0 \leq g(x) \leq 36$$

$$m = 8$$



\*initial values same.  
Group 1 slope > group 2 slope  
Group 1 goes further than group 2.

Ex. 4: An experiment compares the heights of two plants over time. Plant A was 5 cm tall at the beginning and grew 0.3 cm each day. The graph shows the height of plant B in centimeters, as a function of  $t$  days.

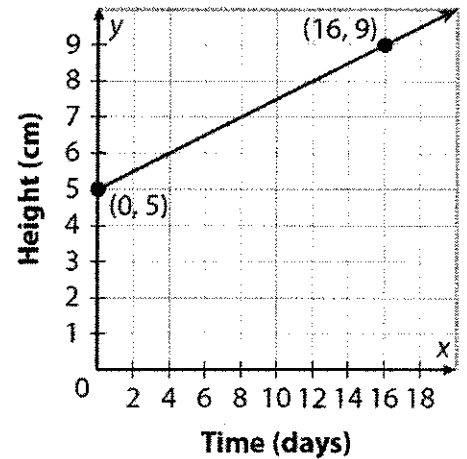
Compare the rates of change and initial values for each.

$$A(x) = .3x + 5$$

$$B(x) = .25x + 5$$

initial values are both 5

rate of change for A > rate of change for B.



Ex. 5 Compare the domain, range, and rates of change of  $f(x)$  and  $g(x)$ . Assume both functions are continuous.

$$f(x):$$

$$D: -1 \leq x \leq 3$$

$$R: -7 \leq f(x) \leq 5$$

$$m = \frac{-4+7}{0+1} = 3$$

Rate of change of  $f(x)$  > rate

$$g(x)$$

$$D: -1 \leq x \leq 3 \text{ Same!}$$

$$R: -4 \leq g(x) \leq 4$$

$$m = 2$$

of change for B

x	f(x)
-1	-7
0	-4
1	-1
2	2
3	5

