

# 3.2 Understanding Relations and Functions



Resource Locker

Essential Question: How do you represent relations and functions?

## Explore Understanding Relations

A **relation** is a set of ordered pairs  $(x, y)$  where  $x$  is the input value and  $y$  is the output value. The **domain** is all possible **inputs** of a relation, and the **range** is all possible **outputs** of a relation. For example, the given relation represents the number of whole-wheat cracker boxes sold and the money earned.

$\{(1, 4), (2, 8), (3, 12), (4, 16)\}$ .

Domain:  $\{1, 2, 3, 4\}$       Range:  $\{2, 8, 12, 16\}$

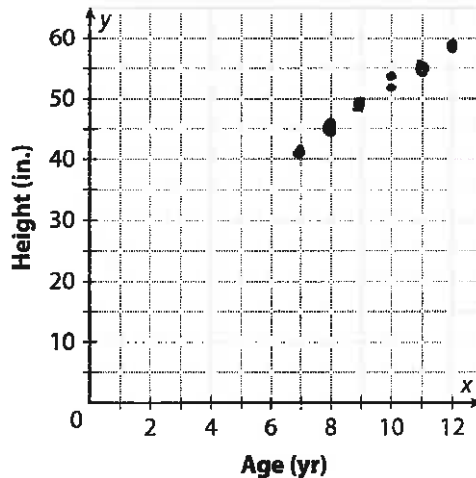
- (A) For the following relation, the input,  $x$ , is the ages of boys and the output,  $y$ , is their corresponding height, in inches.

$\{(7, 41), (8, 45), (9, 49), (10, 52), (10, 53), (11, 55), (12, 59)\}$

- (B) Fill the values in the table.

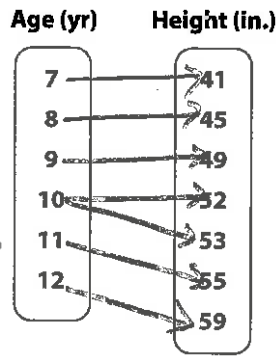
$x$	$y$
7	41
8	45
9	49
10	52
10	53
11	55
12	59

- (C) Plot the points on the graph.



Discrete

D Complete the mapping diagram.



E State the domain of the relation.

$\{7, 8, 9, 10, 11, 12\}$

F State the range of the relation.

$\{41, 45, 49, 52, 53, 55, 59\}$

**Reflect**

1. **Discussion** The number 10 appears twice in the  $x$  column of the table. How many times is it written in the domain? Explain.

Only once. 10 is a possible  $x$ -value. We only write it once.

**Explain 1 Recognizing Functions**

A **function** is a type of relation in which there is only one output value for each input value.

For every input value, there is a unique output value.

Example:  $y = x^2$ . When  $x = 3$ ,  $y$  will always be equal to 9.

**Example 1** Give the domain and range of each relation. State the corresponding outputs for the given inputs in context and explain whether the relation is a function.

A The given relation represents the number of students and the number of classrooms the school has to have for the corresponding number of students.

Students $x$	Classrooms $y$
40	2
45	3
50	4

Domain:  $\{40, 45, 50\}$

The domain represents the number of students.

Range:  $\{2, 3, 4\}$

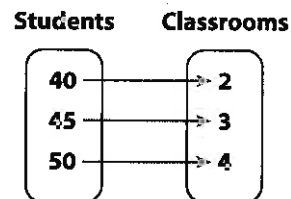
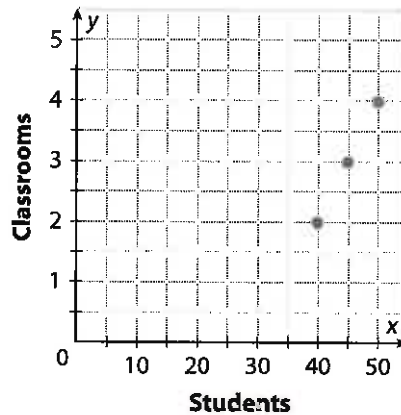
The range represents the number of classrooms.

For an input of 40 students, there is an output of 2 classrooms.

For an input of 45 students, there is an output of 3 classrooms.

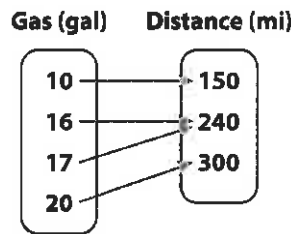
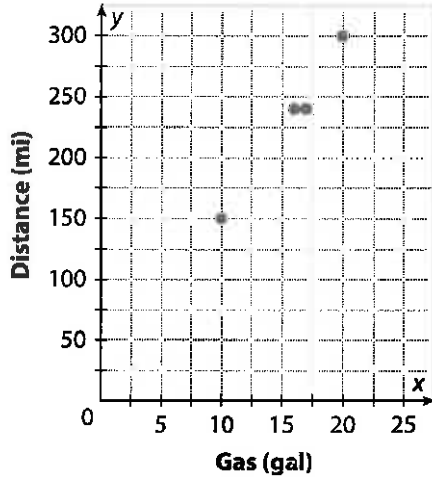
For an input of 50 students, there is an output of 4 classrooms.

This relation is a function. Each domain value is paired with exactly one range value.



- B The given relation represents the amount of gas in gallons and the distance traveled in miles from that amount of gas.

Gas (gal)	Distance (mi)
10	150
16	240
17	240
20	300



Domain:  $\{10, 16, 17, 20\}$

The domain represents Gallons of Gas Used

Range:  $\{150, 240, 300\}$

The range represents Distance Traveled

For an input of 10, there is an output of 150.

For an input of 16, there is an output of 240.

For an input of 17, there is an output of 240.

For an input of 20, there is an output of 300.

This relation is a function. Each domain value is paired with exactly 1 range value.

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**Reflect**

2. If each month in a year was paired with all the possible numbers of days in the month, will the result be a function? Explain.

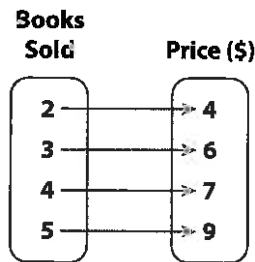
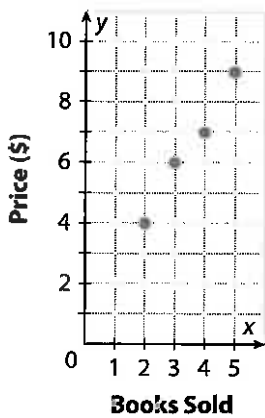
Yes. The domain is the month + no months are repeated in a year

**Your Turn**

Give the domain and range of each relation and interpret them in context. State the corresponding outputs for the given inputs in context and explain whether the relation is a function.

3. The relation represents the number of books sold and the price for the corresponding number of books.

Number of books sold	Price (\$)
2	4
3	6
4	7
5	9



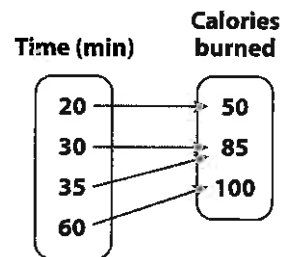
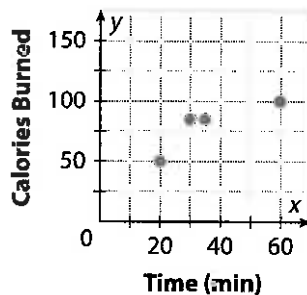
Domain  $\{2, 3, 4, 5\}$   
 Domain is # of books sold

Range  $\{4, 6, 7, 9\}$   
 Range is price

This is a function, as no x-value is paired w/ 2 y values

4. The relation represents the time spent exercising and the number of calories burned during that time.

Time (min)	Calories burned
20	50
30	85
35	85
60	100



Domain  $\{20, 30, 35, 60\}$  Time in minutes spent exercising

Range  $\{50, 85, 100\}$  Calories burned

This is a function

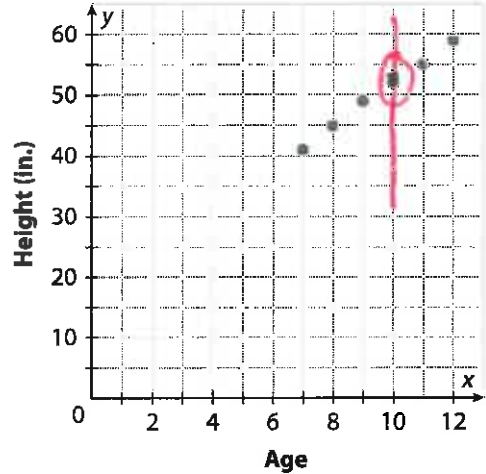
## Explain 2 Understanding the Vertical Line Test

A test, called the *vertical line test*, can be used to determine if a relation is a function. The **vertical line test** states that a relation is a function if and only if a vertical line does not pass through more than one point on the graph of the relation.

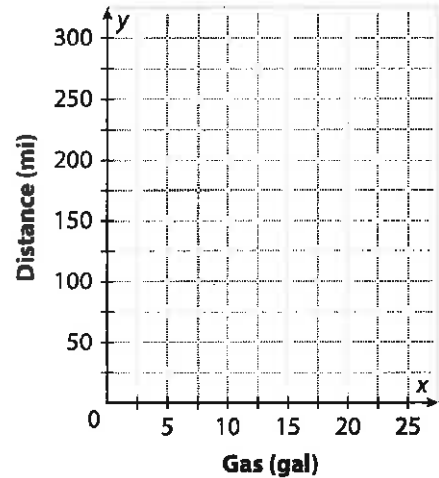
*How does this show it's a function?*

**Example 2** Use the vertical line test to determine if each relation is a function. Explain.

- (A)** Draw a vertical line through each point of the graph.
- Does any vertical line touch more than one point? Yes
- Since a vertical line does pass through more than one point, the graph fails the vertical line test. So, the relation is not a function.



- (B)** Draw a vertical line through each point of the graph.
- Does any vertical line touch more than one point? \_\_\_\_\_
- Since a vertical line \_\_\_\_\_ pass through more than one point, the graph \_\_\_\_\_ the vertical line test. So, the relation \_\_\_\_\_ a function.

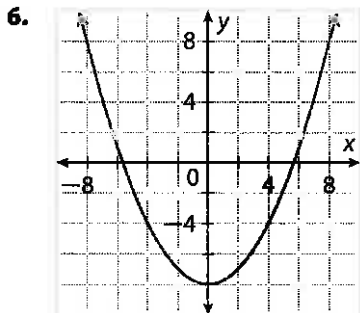


### Reflect

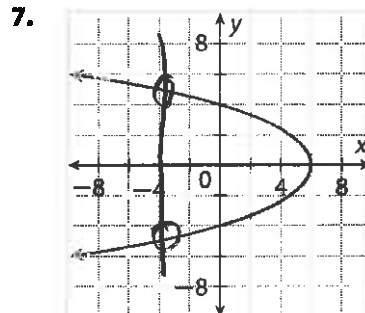
5. Why does the vertical line test work?

### Your Turn

Use the vertical line test to determine if each relation is a function.



function



Not a function

**Elaborate**

8. How can you use a mapping diagram to determine the domain and the range of a relation?

It list the x-values in one column + the y-values in one column w/o repeating

9. **Discussion** For a discrete function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

No, because the input values would be paired w/ more than one output

10. Is a relation a function if its graph intersects the y-axis twice?

No. It would fail the vertical line test

11. **Essential Question Check-In** You are asked to determine if the relation  $y = x^2 - 8x + 4$  is a function. What would be the best way to represent this relation in order to determine if it is a function or not? Explain.

Plug in points + graph

**Evaluate: Homework and Practice**



- Online Homework
- Hints and Help
- Extra Practice

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Express each relation as a table, as a graph, and as a mapping diagram.

1. The relation represents ages of students and the number of words they can write per minute.

$\{(5, 10), (6, 20), (6, 23), (7, 35)\}$

x	y
5	10
6	20
6	23
7	35

