

2.2 Creating and Solving Equations

Essential Question: How do you use an equation to model and solve a real-world problem?



Resource Locker

Explore Creating Equations from Verbal Descriptions

You can use what you know about writing algebraic expressions to write an equation that represents a real-world situation.

Suppose Cory and his friend Walter go to a movie. Each of their tickets costs the same amount, and they share a frozen yogurt that costs \$5.50. The total amount they spend is \$19.90. How can you write an equation that describes the situation?

A Identify the important information.

The word is tells you that the relationship describes an equation.

The word *total* tells you that the operation involved in the relationship is addition.

What numerical information do you have? Yogurt cost \$5.50

Total spent is 19.90

What is the unknown quantity? Cost of the ticket

B Write a verbal description.

Choose a name for the variable. In this case, use *c* for cost of ticket.

The verbal description is: Twice the cost of the ticket plus the cost of yogurt equals total (19.90).

C To write an equation, write a numerical or algebraic expression for each quantity

and insert an equal sign in the appropriate place. An equation is: $2c + 5.50 = 19.90$

Reflect

- How can you use a verbal model to write an equation for the situation described?

2. Could you write a different equation to describe the situation? Explain your reasoning.

Explain 1 Creating and Solving Equations Involving the Distributive Property

When you create an equation to model a real-world problem, your equation may involve the Distributive Property. When you solve a real-world problem, you should always check that your answer makes sense.

Example 1 Write and solve an equation to solve each problem.

- (A) Aaron and Alice are bowling. *Alice depends on Aaron, so let Aaron = a* Alice's score is twice the difference of Aaron's score and 5. The sum of their scores is 320. Find each student's bowling score.



Write a verbal description of the basic situation.

The sum of Aaron's score and Alice's score is 320.

Choose a variable for the unknown quantity and write an equation to model the detailed situation.

Let a represent Aaron's score. Then $2(a - 5)$ represents Alice's score.

$$a + 2(a - 5) = 320$$

Solve the equation for a .

$$a + 2(a - 5) = 320$$

$$a + 2a - 10 = 320 \quad \text{Distributive Property}$$

$$3a - 10 = 320$$

$$3a - \cancel{10} + \cancel{10} = 320 + 10 \quad \text{Addition Property of Equality}$$

$$3a = 330$$

$$\frac{3a}{3} = \frac{330}{3} \quad \text{Division Property of Equality}$$

$$a = 110$$

So, Aaron's score is 110 and Alice's score is $2(a - 5) = 2(110 - 5) = 2(105) = 210$.

Check that the answer makes sense.

$110 + 210 = 320$, so the answer makes sense.

- (B) Mari, Carlos, and Amanda collect stamps. Carlos has five more stamps than Mari, and Amanda has three times as many stamps as Carlos. Altogether, they have 100 stamps. Find the number of stamps each person has.

Write a verbal description of the basic situation.

$$\text{Amanda's stamps} + \text{Carlos's stamps} + \text{Mari's stamps} = 100$$

** Carlos depends on Mari
Amanda depends on Carlos.
Let $s = \#$ of Mari's stamps*

Choose a variable for the unknown quantity and write an equation to model the detailed situation.

Let s represent the number of stamps Mari has. Then Carlos has $s+5$ stamps, and Amanda has $3(s+5)$ stamps.

$$s + s + 5 + 3(s + 5) = 100$$

Solve the equation for s .

$$s + s + 5 + 3(s + 5) = 100$$

$$s + s + 5 + 3s + 15 = 100$$

Distributive Property

$$5s + 20 = 100$$

Combine like terms

$$5s = 80$$

Subtraction Property of Equality

$$s = 16$$

Division Property of Equality

So, Mari has 16 stamps, Carlos has $16+5=21$ stamps, and Amanda has $3(16+5)=63$ stamps.

Check that the answer makes sense.

$$16 + 21 + 63 = 100$$

; the answer makes sense.

$$100 = 100 \checkmark$$

Reflect

3. Would a fractional answer make sense in this situation?

No, because you cannot have a fraction of a stamp

4. **Discussion** What might it mean if a check revealed that the answer to a real-world problem did not make sense?

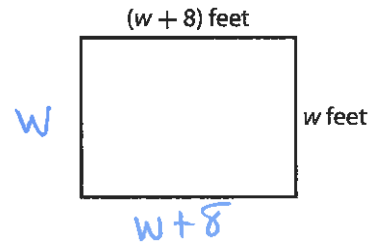
Your Turn

Write and solve an equation to solve the problem.

5. A rectangular garden is fenced on all sides with 256 feet of fencing. The garden is 8 feet longer than it is wide. Find the length and width of the garden.

$$\begin{aligned}
 2w + 2(w + 8) &= 256 \\
 2w + 2w + 16 &= 256 \\
 4w &= 240 \\
 w &= 60
 \end{aligned}$$

width = 60 ft.
length = 68 ft.



Explain 2 Creating and Solving Equations with Variables on Both Sides

In some equations, variables appear on both sides. You can use the properties of equality to collect the variable terms so that they all appear on one side of the equation.

Example 2 Write and solve an equation to solve each problem.

- A** Janine has job offers at two companies. One company offers a starting salary of \$28,000 with a raise of \$3000 each year. The other company offers a starting salary of \$36,000 with a raise of \$2000 each year. In how many years would Janine's salary be the same with both companies? What will the salary be?

Write a verbal description of the basic situation.

Let n represent the number of years it takes for the salaries to be equal.

Base Salary A plus \$3000 per year raise = Base Salary B + \$2000 per year raise

$$28,000 + 3000n = 36,000 + 2,000n$$

$$28,000 + 3000n - 2000n = 36,000 + 2,000n - 2000n \quad \text{Subtraction Property of Equality}$$

$$28,000 + 1000n = 36,000 \quad \text{Combine like terms.}$$

$$28,000 + 1000n - 28,000 = 36,000 - 28,000 \quad \text{Subtraction Property of Equality}$$

$$1000n = 8000$$

$$\frac{1000n}{1000} = \frac{8000}{1000} \quad \text{Division Property of Equality}$$

$$n = 8$$

$$28,000 + 3,000(8) = 36,000 + 2,000(8)$$

$$52,000 = 52,000$$

In 8 years, the salaries offered by both companies will be \$52,000.

Skip in Class

- (B) One moving company charges \$800 plus \$16 per hour. Another moving company charges \$720 plus \$21 per hour. At what number of hours will the charge by both companies be the same? What is the charge?

Company 1: $800 + 16t$
 Company 2: $720 + 21t$

→ Set company charges equal!

Write a verbal description of the basic situation. Let t represent the number of hours that the move takes.

Moving Charge A plus \$16 per hour = Moving Charge B plus \$21 per hour

$$800 + 16t = 720 + 21t$$

$$800 + \cancel{16t} - \cancel{16t} = 720 + 21t - 16t \quad \text{Subtraction Property of Equality}$$

$$800 = 720 + 5t$$

$$800 - 720 = \cancel{720} + 5t - \cancel{720} \quad \text{Subtraction Property of Equality}$$

$$80 = 5t$$

$$\frac{80}{5} = \frac{5t}{5}$$

Division Property of Equality

$$t = 16$$

The charges are the same for a job that takes 16 hours.

Substitute the value 16 in the original equation.

$$800 + 16t = 720 + 21t$$

$$800 + 16(16) = 720 + 21(16)$$

$$800 + 256 = 720 + 336$$

$$1056 = 1056$$

After 16 hours, the moving charge for both companies will be \$1,056.

Reflect

6. Suppose you collected the variable terms on the other side of the equal sign to solve the equation. Would that affect the solution?

Your Turn

Write and solve an equation to solve each problem.

7. Claire bought just enough fencing to enclose either a rectangular garden or a triangular garden, as shown. The two gardens have the same perimeter. How many feet of fencing did she buy? *Set perimeters equal!*

$P_1 = 2(x-3) + 2(3x-3)$
 $2(x-3) + 2(3x-3) = 2(2x-1) + 2x$
 $2x - 6 + 6x - 6 = 4x - 2 + 2x$
 $-12 + 8x = 6x - 2$
 $2x = -2 + 10$
 $2x = 8 \rightarrow x = 4 \rightarrow P = 2(2 \cdot 4 - 1) + 2(4) = 22 \text{ ft.}$

8. A veterinarian is changing the diets of two animals, Simba and Cuddles. Simba currently consumes 1200 Calories per day. That number will increase by 100 Calories each day. Cuddles currently consumes 3230 Calories a day. That number will decrease by 190 Calories each day. The patterns will continue until both animals are consuming the same number of Calories each day. In how many days will that be? How many Calories will each animal be consuming each day then?

Let d = # of days
Simba calories = 1200 + 100d
Cuddles calories = 3230 - 190d
 $1200 + 100d = 3230 - 190d$
 $290d = 2030$
 $d = 7 \text{ days}$
7 days

Explain 3 Constructing Equations from an Organized Table

You can use a table to organize information and see relationships.

Example 3 Construct and solve an equation to solve the problem.

Kim works 4 hours more each day than Jill does, and Jack works 2 hours less each day than Jill does. Over 2 days, the number of hours Kim works is equal to the difference of 4 times the number of hours Jack works and the number of hours Jill works. How many hours does each person work each day?

Analyze Information

Identify the important information.

- Kim works 4 hours more per day than Jill does.
- Jack works 2 hours less per day than Jill does.

Formulate a Plan

Make a table using the information given. Let x be the number of hours Jill works in one day.

	Hours Worked Per Day	Hours Worked Over 2 Days
Kim	$x + 4$	$2(x + 4)$
Jill	x	$2x$
Jack	$x - 2$	$2(x - 2)$

Over 2 days, the number of hours Kim works is equal to the difference of 4 times the number of hours Jack works and the number of hours Jill works.

$2(x+4) = 4(2(x-2)) - 2x$

Solve

$$2(x + 4) = 4 \cdot 2(x - 2) - 2x$$

$$2(x + 4) = 8(x - 2) - 2x$$

Simplify.

$$2x + 8 = 8x - 16 - 2x$$

Distributive Property

$$2x + 8 = 6x - 16$$

$$2x + 8 + 16 = 6x - 16 + 16$$

Addition Property of Equality

$$2x + 24 = 6x$$

$$2x + 24 - 2x = 6x - 2x$$

Subtraction Property of Equality

$$24 = 4x$$

$$\frac{24}{4} = \frac{4x}{4}$$

Division Property of Equality

$$6 = x$$

Jill works 6 hours per day, Kim works 10 hours per day, and Jack works 4 hours per day.

Justify and Evaluate

Substitute $x = 6$ into the original equation.

$$2(6 + 4) = 4 \cdot 2(6 - 2) - 2x$$

$$2(10) = 8(4) - 2(6)$$

$$20 = 20 \checkmark$$

Your Turn

Write and solve an equation to solve the problem.

9. Lisa is 10 centimeters taller than her friend Ian. Ian is 14 centimeters taller than Jim. Every month, their heights increase by 2 centimeters. In 7 months, the sum of Ian's and Jim's heights will be 170 centimeters more than Lisa's height. How tall is Ian now?

	Current height	height in 7 mo
Lisa	$j + 14 + 10 \rightarrow j + 24$	$j + 24 + 7(2) \rightarrow j + 38$
Ian	$j + 14$	$j + 14 + 7(2) \rightarrow j + 28$
Jim	j	$j + 7(2) \rightarrow j + 14$

$$j + 28 + j + 14 = 170 + j + 38$$

$$2j + 42 = j + 208$$

$$j = 166$$

Jim is 166 cm, so
Ian is 180 cm

Elaborate

10. How can you use properties to solve equations with variables on both sides?

11. How is a table helpful when constructing equations?

12. When solving a real-world problem to find a person's age, would a negative solution make sense? Explain.

13. **Essential Question Check-in** How do you write an equation to represent a real-world situation?

Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Write an equation for each description.

1. The sum of 14 and a number is equal to 17.

$$14 + n = 17$$

2. A number increased by 10 is 114.

$$n + 10 = 114$$

3. The difference between a number and 12 is 20.

$$n - 12 = 20$$

4. Ten times the sum of half a number and 6 is 8.

$$10 \left(\frac{1}{2}n + 6 \right) = 8$$

5. Two-thirds a number plus 4 is 7.

$$\frac{2}{3}n + 4 = 7$$

6. Tanmayi wants to raise \$175 for a school fundraiser. She has raised \$120 so far. How much more does she need to reach her goal?

$$175 - 120 = x$$

7. Hector is visiting a cousin who lives 350 miles away. He has driven 90 miles. How many more miles does he need to drive to reach his cousin's home?

$$90 + x = 350$$

8. The length of a rectangle is twice its width. The perimeter of the rectangle is 126 feet.

