

2.1 Modeling with Expressions



Resource Locker

Essential Question: How do you interpret algebraic expressions in terms of their context?

Explore Interpreting Parts of an Expression

An **expression** is a mathematical phrase that contains operations, numbers, and/or variables. The **terms** of an expression are the parts that are being added. A **coefficient** is the numerical factor of a variable term. There are both *numerical expressions* and *algebraic expressions*. A **numerical expression** contains only numbers while an **algebraic expression** contains at least one variable.

(A) Identify the terms and the coefficients of the expression $8p + 2q + 7r$.
 terms: $8p, 2q, 7r$; coefficients: $8, 2, 7$

(B) Identify the terms and coefficients of the expression $18 - 2x - 4y$. Since the expression involves Subtraction rather than addition, rewrite the expression as the Sum of the terms:
 $18 - 2x - 4y = \underline{18 + (-2x) + (-4y)}$. So, the terms of the expression are $18, -2x, -4y$ and the coefficients are $-2, -4$

(C) Identify the terms and coefficients in the expression $2x + 3y - 4z + 10$. Since the expression involves both Subtraction and addition, rewrite the expression as the Sum of the terms: $2x + 3y - 4z + 10 = \underline{2x + 3y + (-4z) + 10}$. So, the terms of the expression are $2x, 3y, -4z$ and the coefficients are $2, 3, -4$

Tickets to an amusement park are \$60 for adults and \$30 for children. If a is the number of adults and c is the number of children, then the cost for a adults and c children is $60a + 30c$.

(D) What are the terms of the expression? $60a, 30c$

(E) What are the factors of $60a$? $60, a$

(F) What are the factors of $30c$? $30, c$

(G) What are the coefficients of the expression? $60, 30$

(H) Interpret the meaning of the two terms of the expression. $60a$ is the cost of all the adults; $30c$ is the cost of all the children

The price of a case of juice is \$15.00. Fred has a coupon for 20 cents off each bottle in the case. The expression to find the final cost of the case of juice is $15 - 0.2b$, wherein b is the number of bottles.



(I) What are the terms of the expression? $15, -0.2b$

- J) What are the factors of each term? 15 is the only factor of the constant (15) term and -0.2 and b are the factors of the -0.2b term.
- K) Do both terms have coefficients? Explain. No, because 15 does not have a variable What are the coefficients? -0.2
- L) What does the expression $15 - 0.2b$ mean in the given situation?
The final cost of the juice after the discount

Reflect

- Sally identified the terms of the expression $9a + 4b - 18$ as $9a$, $4b$, and 18 . Explain her error.
- What is the coefficient of b in the expression $b + 10$? Explain.

Explain 1 Interpreting Algebraic Expressions in Context

In many cases, real-world situations and algebraic expressions can be related. The coefficients, variables, and operations represent the given real-world context.

Interpret the algebraic expression corresponding to the given context.

Example 1

- A) Curtis is buying supplies for his school. He buys p packages of crayons at \$1.49 per package and q packages of markers at \$3.49 per package. What does the expression $1.49p + 3.49q$ represent?

Interpret the meaning of the term $1.49p$. What does the coefficient 1.49 represent?

The term $1.49p$ represents the cost of p packages of crayons. The coefficient represents the cost of one package of crayons, \$1.49.

Interpret the meaning of the term $3.49q$. What does the coefficient 3.49 represent?

The term $3.49q$ represents the cost of q packages of markers. The coefficient represents the cost of one package of markers, \$3.49.

Interpret the meaning of the entire expression.

The expression $1.49p + 3.49q$ represents the total cost of p packages of crayons and q packages of markers.

- B) Jill is buying ink jet paper and laser jet paper for her business. She buys 8 more packages of ink jet paper than p packages of laser jet paper. Ink jet paper costs \$6.95 per package and laser jet paper costs \$8 per package. What does the expression $8p + 6.95(p + 8)$ represent?

Interpret the meaning of the first term, $8p$. What does the coefficient 8 represent?

The term $8p$ represents the cost of all packs of laser paper. The coefficient represents \$8 cost per pack, \$8.

Interpret the meaning of the second expression, $6.95(p + 8)$. What do the factors 6.95 and $(p + 8)$ represent?

The term $6.95(p + 8)$ represents the total cost of the ink-jet paper. 6.95 represents the cost of each pack of ink-jet. $(p + 8)$ represents the number of packs of ink-jet paper that Jill bought.

Interpret the expression $8p + 6.95(p + 8)$.

The expression represents the total cost of ALL the paper that Jill bought.

Your Turn

Interpret the algebraic expression corresponding to the given context.

- George is buying watermelons and pineapples to make fruit salad. He buys w watermelons at \$4.49 each and p pineapples at \$5 each. What does the expression $4.49w + 5p$ represent?
- Sandi buys 5 fewer packages of pencils than p packages of pens. Pencils costs \$2.25 per package and pens costs \$3 per package. What does the expression $3p + 2.25(p - 5)$ represent?

Explain 2 Comparing Algebraic Expressions

Given two algebraic expressions involving two variables, we can compare whether one is greater or less than the other. We can denote the inequality between the expressions by using $<$ or $>$ symbols. If the expressions are the same, or **equivalent expressions**, we denote this equality by using $=$.

Suppose x and y give the populations of two different cities where $x > y$. Compare the expressions and tell which of the given pair is greater.

Example 1

- (A) $x + y$ and $2x$

**Can also plug in concrete numbers to test, like $x=4$ and $y=2$*

The expression $2x$ is greater.

- Putting the lesser population, y , together with the greater population, x , gives a population that is less than double the greater population.

- (B) $\frac{x}{y}$ and $\frac{y}{x}$

Since $x > y$, $\frac{x}{y}$ will be greater than 1 and $\frac{y}{x}$ will be less than 1.

So $\frac{x}{y} > \frac{y}{x}$.

Your Turn

Suppose x and y give the populations of two different cities where $x > y$ and $y > 0$. Compare the expressions and tell which of the given pair is greater.

5. $\frac{x}{x+y}$ and $\frac{x+y}{x}$ *→ bigger value on top*

bigger value on bottom
 $\frac{x}{x+y} < \frac{x+y}{x}$

6. $2(x+y)$ and $(x+y)^2$
 $(x+y) + (x+y)$ $(x+y) \cdot (x+y)$
 $2(x+y) < (x+y)^2$

Explain 3 Modeling Expressions in Context

The table shows some words and phrases associated with the four basic arithmetic operations. These words and phrases can help you translate a real-world situation into an algebraic expression.

Operation	Words	Examples
Addition	the sum of, added to, plus, more than, increased by, total, altogether, and	1. A number increased by 2 2. The sum of n and 2 3. $n + 2$
Subtraction	less than, minus, subtracted from, the difference of, take away, taken from, reduced by	1. The difference of a number and 2 2. 2 less than a number 3. $n - 2$
Multiplication	times, multiplied by, the product of, percent of	1. The product of 0.6 and a number 2. 60% of a number 3. $0.6n$
Division	divided by, division of, quotient of, divided into, ratio of,	1. The quotient of a number and 5 2. A number divided by 5 3. $n \div 5$ or $\frac{n}{5}$

Example 3 Write an algebraic expression to model the given context. Give your answer in simplest form.

(A) the price of an item plus 6% sales tax

$$\begin{array}{rcl} \text{Price of an item} & + & \text{6\% sales tax} \\ p & + & 0.06p \end{array}$$

The algebraic expression is $p + 0.06p$, or $1.06p$.

(B) the price of a car plus 8.5% sales tax

$$\begin{array}{rcl} \text{The price of a car} & + & \text{8.5\% sales tax} \\ p & + & .085p \end{array}$$

The algebraic expression is $p + .085p$
or $1.085p$.

Reflect

7. Use the Distributive Property to show why $p + 0.06p = 1.06p$.

8. What could the expression $3(p + 0.06p)$ represent? Explain.

Your Turn

Write an algebraic expression to model the given context. Give your answer in simplest form.

9. the number of gallons of water in a tank, that already has 300 gallons in it, after being filled at 35 gallons per minute for m minutes

$$300 + 35m$$

10. the original price p of an item less a discount of 15%

$$p - .15p$$