

13.3 Combinations

Starter:

A. List all the possible permutations (different orders) of just 3 letters that you can choose from A B C D. Then, use the permutation formula to see how many order options

(permutations) you should have. ~~ABC~~ ~~ACB~~ ~~BAC~~ ~~BCA~~ ~~CAB~~ ~~CBA~~
 $4^P_3 = \frac{4!}{(4-3)!} = 24$ ACD ADC CAD CDA DAC DCA
 BCD BDC CBD CDB DCB DCB
 BAD BDA ABD ADB DAB DBA

B. Now, let's consider what would happen if the order of the letters doesn't matter. Cross out all duplicate groups of letters. For example, if you have ABC and ACB, and BCA, these are all the same group of letters, so cross out all but one. How many different combinations remain? 4

Basically, you divided the number of permutation by 6 or 3!.

A Combination is a selection of objects in which order is not important. For example, a group of three boys- John, Joe, and Jarod - is the same as the group of Joe, Jarod, and John.

The number of combinations of n objects, choosing r at a time, is: ${}_nC_r = \frac{n!}{(n-r)!r!}$

Evaluate each:

<p>1. ${}_5C_5$ $\frac{5!}{(5-5)!5!} = 1$</p>	<p>2. ${}_{15}C_8$ $\frac{15!}{(15-8)!8!} = \frac{15!}{7!8!}$ 6435</p>	<p>3. ${}_{15}P_8$ $\frac{15!}{(15-8)!} = \frac{15!}{7!}$ 259,459,200</p>
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Determine whether each situation is a Permutation or Combination and solve.

<p>4. Four students will be chosen from a class of 120 to lead a fundraiser. How many groups of 4 students are possible? ${}_{120}C_4 = \frac{120!}{(120-4)!4!} = \frac{120!}{116!4!}$ <u>4,845 groups</u> Combination</p>	<p>5. Ten students are auditioning for 3 roles in a play. In how many ways can the 3 roles be filled? ${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!}$ <u>720 ways</u> Permutation</p>
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Using Counting Principle AND combinations

6. You are ordering a burrito with 2 main ingredients and 3 toppings from the menu to the right. How many different burritos are possible?

MAIN INGREDIENTS		TOPPING'S	
chicken	black beans	peppers	salsa
steak	red beans	onions	pico de gallo
pork	rice	guacamole	extra cheese
		jalapeños	black olives

$$\frac{{}^6C_2 \cdot {}^8C_3}{\frac{6!}{4! \cdot 2!} \cdot \frac{8!}{5! \cdot 3!}} = 15 \cdot 56 = 840 \text{ burritos}$$

Homework:

Evaluate each:

1. 8P_3 $\frac{8!}{(8-3)!} = \frac{8!}{5!}$ 336	2. $\frac{12!}{4!8!} = 495$	3. 9C_6 $\frac{9!}{(9-6)!6!} = \frac{9!}{3!6!} = 84$
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Determine whether each situation is a permutation or combination. Then answer each:

4. Of the roughly 50 female members of the senior class, only 6 are being chosen for the prom court. How many possible groups of girls could be chosen for the court? ${}_{50}C_6 = \frac{50!}{(50-6)!6!} = \frac{50!}{44!6!}$ 15,890,700 groups Combination	5. Jerome is going to sponsor a new bowling team. Only 4 members can bowl at a time out of the 6 member team. How many different line-ups are possible? ${}_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$ 360 line-ups Permutation
6. You just bought season 1 of your favorite show, which contains 13 episodes. You have time to watch only 3 episodes. How many groups of 3 episodes can be chosen? ${}_{13}C_3 = \frac{13!}{(13-3)!3!} = \frac{13!}{10!3!}$ 286 groups Combination	7. You are dealt a hand of 5 cards from a standard deck of cards. In how many ways can the cards be arranged in your hand? ${}_{52}P_5 = \frac{52!}{(52-5)!} = \frac{52!}{47!}$ 31,875,200 ways Permutation