

13.2 Permutations

Starter: $n!$ means you take n and multiply it by all the numbers below it.

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ $0! = 1$

Find:

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$$

$$\frac{12!}{8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!}} = 12 \cdot 11 \cdot 10 \cdot 9 = 11,880$$

Permutation: An arrangement of objects in a certain order.

Linear Permutation: The arrangement of objects in a line. The number of arrangements

(permutations) of n number of objects, taken r at a time is found by using $P_{(n,r)} = \frac{n!}{(n-r)!}$

Evaluate each expression:

$$1. P_{(5,5)} = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = 120$$

$$2. P_{(6,4)} = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360$$

$$3. \frac{P_{(6,4)}}{P_{(5,3)}} \rightarrow 360$$

$$P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

$$\frac{6P_4}{5P_3} = \frac{360}{60} = 6$$

Examples:

4. In how many ways can you re-arrange the letters in the word JULY ?

$${}_4P_4 = \frac{4!}{(4-4)!} = 24 \text{ ways}$$

$$\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$$

5. You have 7 seven books, but only 3 will be placed on a given shelf. In how many ways can the 3 books be arranged?

$$\underline{7} \cdot \underline{6} \cdot \underline{5}$$

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$$

6. Your band has written 12 songs, but only 9 will be recorded for a CD. In how many ways can the songs be arranged?

$${}_{12}P_9 = \frac{12!}{(12-9)!} = \frac{12!}{3!}$$

$$79,833,600$$

7. A group of 5 teens enter a movie theater. They found a row with 7 empty seats. In how many ways can the teens be seated in the row?

$${}_7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 2520$$

Homework

Evaluate each of the following:

$$1. \frac{8!}{3!} = \boxed{6,720}$$

$$2. P(8, 6) = \frac{8!}{(8-6)!}$$

$${}_8P_6 = \boxed{20,160}$$

$$3. \frac{P(8,3)}{7!} = \frac{{}_8P_3}{7!} = \frac{8!}{(8-3)! \cdot 7!}$$

$$= \frac{336}{5040}$$

$$= \frac{1}{15}$$

Use a permutation to determine the number of ways you can arrange a) all the letters of the word and b) just 2 of the letters in the word.

4. GAME

$${}_4P_4 = \frac{4!}{0!}$$

$$= \boxed{24}$$

$${}_4P_2 = \frac{4!}{2!}$$

$$= \boxed{12}$$

5. FLORIDA

$${}_7P_7 = \frac{7!}{0!}$$

$$= 5,040$$

$${}_7P_2 = \frac{7!}{5!}$$

$$= 2,520$$

6. Six friends go the movie theater. In how many ways can they sit in a row of 10 empty seats?

$${}_{10}P_6 = \frac{10!}{4!} = \boxed{151,200}$$

7. Twenty students are running for 3 different positions on student council. In how many orders can the 3 positions be filled?

$${}_{20}P_3 = \frac{20!}{17!} = \boxed{6,840}$$

8. There are 11 entries for judging in an art contest. In how many orders can the 1st, 2nd, and 3rd place winners be chosen?

$${}_{11}P_3 = \frac{11!}{(11-3)!} = \frac{11!}{8!}$$

$$= \boxed{990}$$

9. On a math exam, you are asked to list 6 provided steps to solving a particular problem in the correct order. You guess the order of the steps at random. How many options are there to choose from?

$${}_6P_6 = \frac{6!}{0!} = \boxed{720}$$