

11.4 Pythagorean Theorem

A Little Geometry Vocabulary

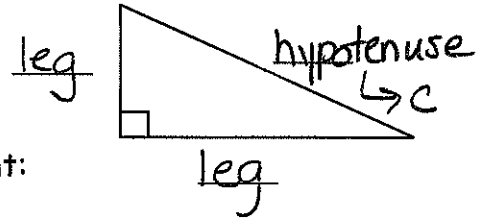
Right Triangle: A triangle containing a right (90°) angle.

Hypotenuse: The longest side of a right triangle, which is opposite the right angle.

Legs: The two sides that form the right angle.

**Label the legs and hypotenuse in the right triangle to the right:

Theorem: a statement that can be proven true.



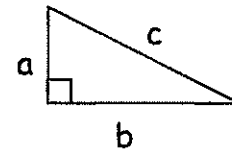
What does the **PYTHAGOREAN THEOREM** say??

In a right triangle, the sum of the squares of the measures of the legs equals the square of the hypotenuse. In other words, $leg^2 + leg^2 = hypotenuse^2$, or

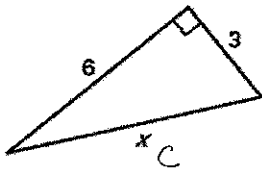
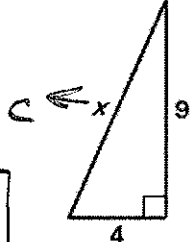
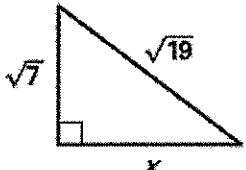
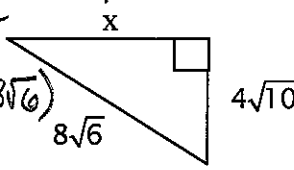
for the triangle at the right, $a^2 + b^2 = c^2$

What makes a triangle right?

If the sum of the squares of the measures of two sides of a triangle equals the square of the longest side, then the triangle is a right triangle. So for 3 sides lengths of a triangle, a, b, and c, where c is the longest side, if $a^2 + b^2 = c^2$, then the triangle is a right triangle



Examples-For each of the following, find the missing length. Write all exact answers-simplified radicals!

| | |
|--|--|
| <p>1. $3^2 + 6^2 = x^2$ $9 + 36 = x^2$ $\sqrt{45} = \sqrt{x^2}$ $x = \pm 3\sqrt{5}$ $x = 3\sqrt{5}$</p>  | <p>2. $4^2 + 9^2 = x^2$ $16 + 81 = x^2$ $97 = x^2$ $\pm\sqrt{97} = x$ $x = \sqrt{97}$</p>  |
| <p>3. $x^2 + (\sqrt{7})^2 = (\sqrt{19})^2$ $x^2 + 7 = 19$ $x^2 = 12$ $x = \pm\sqrt{12}$ $x = 2\sqrt{3}$</p>  | <p>4. $x^2 + (4\sqrt{10})^2 = (8\sqrt{6})^2$ $x^2 + (4\sqrt{10})(4\sqrt{10}) = (8\sqrt{6})(8\sqrt{6})$ $x^2 + 16 \cdot 10 = 64 \cdot 6$ $x^2 + 160 = 384$ $\sqrt{x^2} = \sqrt{224}$ $x = \sqrt{16 \cdot 14}$ $x = 4\sqrt{14}$</p>  |

Determine whether a right triangle can be formed by the given side lengths: $\rightarrow C$

| | | |
|---|--|---|
| <p>5. 6in, 8in, 7in $\rightarrow C$</p> $6^2 + 7^2 \quad 8^2$ $36 + 49 \quad 64$ $85 \neq 64$ <p>Not a right Δ</p> | <p>6. $\sqrt{3}, \sqrt{4}, \sqrt{5} \rightarrow C$</p> $(\sqrt{3})^2 + (\sqrt{4})^2 \stackrel{?}{=} (\sqrt{5})^2$ $3 + 4 \stackrel{?}{=} 5$ $7 \neq 5$ <p>Not a right Δ</p> | <p>7. 9, $3\sqrt{34}$, 15</p> $9^2 + 15^2 \stackrel{?}{=} (3\sqrt{34})^2$ $81 + 225 \stackrel{?}{=} (3\sqrt{34})(3\sqrt{34})$ $306 \stackrel{?}{=} 9 \cdot 34$ $306 = 306 \checkmark$ <p>Right Δ</p> |
|---|--|---|

Homework

The table below shows the sides lengths of 4 different triangles. Find in the missing lengths as simplified radicals and show all work to the right:

| | Leg | Leg | Hypotenuse |
|----|------------|-------------|-------------|
| 1. | 8 | 15 | 17 |
| 2. | 6 | $\sqrt{13}$ | 7 |
| 3. | $\sqrt{5}$ | $\sqrt{15}$ | $2\sqrt{5}$ |
| 4. | 2 | $2\sqrt{3}$ | 4 |

Work:

$$8^2 + 15^2 = c^2 \quad 6^2 + b^2 = 7^2$$

$$64 + 225 = c^2 \quad 36 + b^2 = 49$$

$$289 = c^2 \quad b^2 = 13$$

$$17 = c \quad b = \sqrt{13}$$

$$(\sqrt{5})^2 + (\sqrt{15})^2 = c^2$$

$$5 + 15 = c^2 \quad a^2 + (2\sqrt{3})^2 = 4^2$$

$$\sqrt{20} = \sqrt{c^2} \quad a^2 + (2\sqrt{3})(2\sqrt{3}) = 16$$

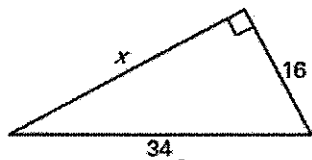
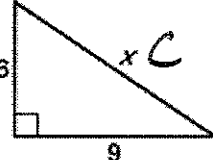
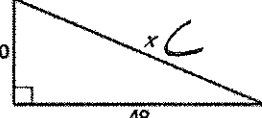
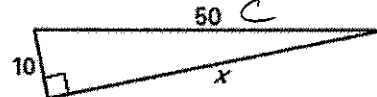
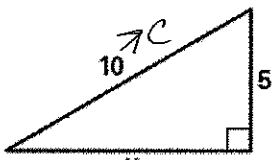
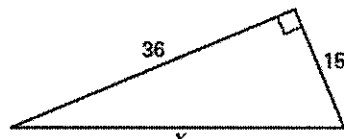
$$2\sqrt{5} = c \quad a^2 + 4 \cdot 3 = 16$$

$$a^2 + 12 = 16$$

$$a^2 = 4$$

$$a = 2$$

In 5-10, identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

| | | |
|---|--|--|
| <p>5. </p> $x^2 + 16^2 = 34^2$ $x^2 + 256 = 1156$ $x^2 = 900$ $x = 30$ | <p>6. </p> $6^2 + 9^2 = x^2$ $36 + 81 = x^2$ $117 = x^2$ $x = \sqrt{117}$ $x = \sqrt{9 \cdot 13}$ $x = 3\sqrt{13}$ | <p>7. </p> $20^2 + 48^2 = x^2$ $400 + 2304 = x^2$ $2704 = x^2$ $52 = x$ |
| <p>8. </p> $x^2 + 10^2 = 50^2$ $x^2 + 100 = 2500$ $x^2 = 2400$ $x = \sqrt{2400}$ $x = \sqrt{400 \cdot 6}$ $x = 20\sqrt{6}$ | <p>9. </p> $x^2 + 5^2 = 10^2$ $x^2 + 25 = 100$ $x^2 = 75$ $x = \sqrt{75}$ $x = 5\sqrt{3}$ | <p>10. </p> $15^2 + 36^2 = x^2$ $225 + 1296 = x^2$ |